MOHIDJET – Technical manual

Objective

This manual aims to describe the theory behind the MOHIDJET.

Overview

The MOHIDJET integral model aims to simulate the initial dilution associated to outfalls jets. The model is used as an initial condition of the MOHID (http://www.mohid.com) system Lagrangian tracers module. The MOHIDJET is a very helpful tool to simulate the impact of outfalls water bodies integrating the near field (MOHIDJET) with the far field (MOHID).

A Lagrangian approach was used in the MOHIDJET similar to the one use in the JETLAG model (Lee and Cheung, 1990, http://www.aoe-water.hku.hk/visjet/index.htm). Basically is simulated the trajectory and volume variation of a tracer with a cylindrical geometry. However, for the entrainment parametrization was used the work of Jirka (1999). This author is one of the main contributors to the development of CORJET (Cornell Buoyant Jet Integral Model) the buoyant jet model of CORMIX (Cornell Mixing Zone Expert System).

Description

Discharge Properties

The properties presented in the Table 1 are used to describe the jet properties at the port exit.

Table 1 – Jet properties at the port exit.

Property	**Description**
Δh_0	The length of the emitted "cylinder"
D_0	Port diameter
Q_0	Port flow
ρ_0	Jet Density at the Port exit
θ_0	Angle between the port normal vector and the horizontal plane
σ_0	Angle between the port normal vector projection in the horizontal plane and the x direction

Forces

In the case of a jet emitted in a still water body with no impulsion no force is applied to the jet. However, if the jet density is different form the ambient than impulsion must be consider (I). Impulsion has only a vertical component.

$$\vec{I} = g(\rho_{Ambient} - \rho_{Jet})V_{Tracer}\vec{e}_z$$

Another force is present if there is an ambient flow. In this case if the projection of ambient velocity in plane normal to the jet trajectory is different from zero than a drag force is present (Figure 1).

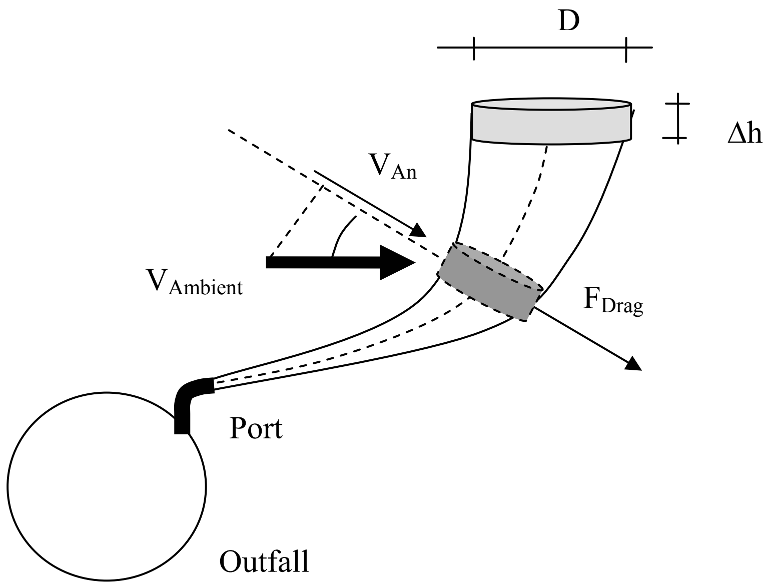


Figure 1 – Drag force acting over the plume.

This drag force is similar to the drag force establish around a cylindrical body (1).

$$\overrightarrow{F_D} = C_D \frac{1}{2} \rho_{Ambient} D \Delta h \left|\overrightarrow{V_{An}}\right| \overrightarrow{V_{An}} \tag{1}$$

All the variables are known except the drag coefficient. Jirka (1999) consider this parameter equal to 1.3. The projection of ambient velocity in the plane normal to the tracer velocity ($\left(\overrightarrow{V_{An}}\right)$ can be easily computed. If $\vec{n}$ (n_x,n_y,n_z) is the unitary vector of the tracer velocity than:

$$\vec{V}_{Ambient} \cdot \vec{n} = \left|\vec{V}_{Ambient}\right| \cdot |\vec{n}| \cos(\varpi) = \left|\vec{V}_{Ambient}\right| \cos(\varpi) \Leftrightarrow$$

$$\varpi = \arccos\left(\frac{\vec{V}_{Ambient} \cdot \vec{n}}{\left|\vec{V}_{Ambient}\right|}\right) \tag{2}$$

$$\vec{V}_{An} = \vec{V}_{Ambient} \cos(\psi) \wedge \psi = 90 - \varpi$$

$$\vec{V}_{An} = \vec{V}_{Ambient} \cos(90 - \varpi)$$

The unitary vector of the tracer velocity can be compute from the tracer velocity in the follow way:

$$\vec{V}_{Tracer} = \left(u_{Tracer}, v_{Tracer}, w_{Tracer}\right)$$

$$\theta_T = \begin{cases} \arctan\left(\dfrac{w_{Tracer}}{\sqrt{u_{Tracer}{}^2 + v_{Tracer}{}^2}}\right) \wedge u_{Tracer} > 0 \\ \arctan\left(\dfrac{w_{Tracer}}{\sqrt{u_{Tracer}{}^2 + v_{Tracer}{}^2}}\right) + \pi \wedge u_{Tracer} < 0 \end{cases} \quad (3)$$

$$\sigma_T = \begin{cases} \arctan\left(\dfrac{v_{Tracer}}{u_{Tracer}}\right) \wedge u_{Tracer} > 0 \\ \arctan\left(\dfrac{v_{Tracer}}{u_{Tracer}}\right) + \pi \wedge u_{Tracer} < 0 \end{cases} \quad (4)$$

$$\vec{n} = \left(\cos(\Theta_T)\cos(\sigma_T), \cos(\Theta_T)\sin(\sigma_T), \sin(\Theta_T)\right)$$

Along the drag force direction the jet plume velocity is null. The drag force can have components different from zero in the 3 cartesian directions (x,y,z). In this case is necessary to take in account the effect of this force in the three components of the tracer velocity.

Properties evolution

The MOHIDJET computes the evolution of the several properties of a cylindrical tracer in a stationary environment. In Table 2 the properties simulated are presented.

Table 2 – List of properties computed along the jet plume.

Tracer	Name variable	Dominant Processes	Equation (or law)
Velocity	$\vec{V}_{Tracer}$	• Volume variation • Impulsion • Drag force	Momentum conservation
Volume	V_{Tracer}	• Shear entrainment • Drag entrainment	Mass conservation
Length	Δh_{Tracer}	Velocity divergence	Mass conservation
Salinity Temperature	S_{Tracer} T_{Tracer}	Volume variation	Mass and heat conservation
Density	ρ_{Tracer}	Volume variation	UNESCO equation

Volume Variation

The volume variation is controlled manly by 2 processes shear entrainment and entrainment associated with the force drag describe earlier. The first process is proportional to the difference between the tracer velocity and ambient velocity component parallel to the plume trajectory. The second process is proportional to the projection of the ambient velocity in plane normal to the plume trajectory. Jirka (1999) present the follow formulation:

$$\frac{\partial V_{Tracer}}{\partial t} = E_S + E_D$$

$$E_S = \alpha_S \pi D \Delta h \left| \vec{V}_{Tracer} - \vec{V}_{Ambient} \cdot \vec{n} \right| \qquad (5)$$

$$E_D = \alpha_D \pi D \Delta h \left| \vec{V}_{An} \right|$$

$$\alpha_S = 0.055 + \frac{0.6\sin(\theta_{Tracer})}{Fl^2} + 0.055 \frac{\left|\vec{V}_{Ambient} \cdot \vec{n}\right|}{\left|\vec{V}_{Tracer}\right|} \wedge Fl^2 = \max\left(4.66^2, \frac{\left|\vec{V}_{Tracer}\right|}{gD\frac{\rho_{Ambient} - \rho_{tracer}}{\rho_{Ambient}}}\right)$$

$$\alpha_D = 0.5$$

The discretization of (5) is:

$$V^{t+\Delta t} = V^t + \left(E_S + E_D\right)\Delta t$$

Density Variation

In a first approach a simple density variation due to entrainment was considered. In the future salinity and temperature variations due to entrainment will be computed and density is obtained using the UNESCO equation. This equation relates temperature and salinity with density.

The equation presently use is:

$$\frac{\partial \rho_T V_T}{\partial t} = \left(E_S + E_D\right)\rho_{Ambient} \qquad (6)$$

The descrization of equation (6):

$$\rho_{Tracer}^{t+\Delta t} V_{Tracer}^{t+\Delta t} = \rho_{Tracer}^{t} V_{Tracer}^{t} + \rho_{Ambient}\left(E_S + E_D\right)\Delta t = \rho_{Tracer}^{t} V_{Tracer}^{t} + \rho_{Ambient}\Delta V_{Tracer} \Leftrightarrow$$

$$\rho_{Tracer}^{t+\Delta t} = \frac{\rho_{Tracer}^{t} V_{Tracer}^{t} + \rho_{Ambient}\Delta V_{Tracer}}{V_{Tracer}^{t+\Delta t}}$$

Velocity Variation

To compute the tracer velocity evolution the law of momentum conservation is used. This law says that the sum of forces applied to a body (in this case the cylindrical tracer) is equal to is acceleration (7).

$$\frac{\partial M\vec{V}_{Tracer}}{\partial t} = \vec{I} + \vec{F}_D \qquad (7)$$

The discretization is presented by components.

$$\frac{\left(Mu_{Tracer}\right)^{t+\Delta t}-\left(Mu_{Tracer}\right)^{t}}{\Delta t}=F_{Dx}$$
$$\frac{\left(Mv_{Tracer}\right)^{t+\Delta t}-\left(Mv_{Tracer}\right)^{t}}{\Delta t}=F_{Dy}$$
$$\frac{\left(Mw_{Tracer}\right)^{t+\Delta t}-\left(Mw_{Tracer}\right)^{t}}{\Delta t}=F_{Dz}+I$$

The new velocities can be computed in the follow way:

$$u_{Tracer}^{t+\Delta t}=\frac{\rho_{Tracer}^{t}V_{Tracer}^{t}u_{Tracer}^{t}+\rho_{Ambient}\Delta Vu_{Ambient}+F_{Dx}\Delta t}{\rho_{Tracer}^{t+\Delta t}V_{Tracer}^{t+\Delta t}}=$$
$$\frac{\rho_{Tracer}^{t}V_{Tracer}^{t}u_{Tracer}^{t}+\rho_{Ambient}\Delta Vu_{Ambient}+C_{D}\frac{1}{2}\rho_{Ambient}D\Delta h\left|\overrightarrow{V_{An}}\right|V_{Anx}\Delta t}{\rho_{Tracer}^{t+\Delta t}V_{Tracer}^{t+\Delta t}} \quad \textbf{(8)}$$

$$v_{Tracer}^{t+\Delta t}=\frac{\rho_{Tracer}^{t}V_{Tracer}^{t}v_{Tracer}^{t}+\rho_{Ambient}\Delta Vv_{Ambient}+C_{D}\frac{1}{2}\rho_{Ambient}D\Delta h\left|\overrightarrow{V_{An}}\right|V_{Any}\Delta t}{\rho_{Tracer}^{t+\Delta t}V_{Tracer}^{t+\Delta t}} \quad \textbf{(9)}$$

$$w_{Tracer}^{t+\Delta t}=\frac{\rho_{Tracer}^{t}V_{Tracer}^{t}w_{Tracer}^{t}+\rho_{Ambient}\Delta Vw_{Ambient}+C_{D}\frac{1}{2}\rho_{Ambient}D\Delta h\left|\overrightarrow{V_{An}}\right|V_{Anz}\Delta t}{\rho_{Tracer}^{t+\Delta t}V_{Tracer}^{t+\Delta t}}+ \quad \textbf{(10)}$$
$$g\frac{\left(\rho_{Ambient}-\rho_{Tracer}^{t+\Delta t}\right)}{\rho_{Tracer}^{t+\Delta t}}\Delta t$$

Tracer (cylindrical) length variation

The cylindrical tracer suffers compression and stretching due to advective acceleration. It’s possible to estimate the tracer length variation looking to the variability of the tracer velocity along the plume trajectory.

$$\frac{\partial\Delta h}{\partial t}=\left(\vec{V}_{Tracer}\right)_{x,y,z}-\left(\vec{V}_{Tracer}\right)_{x-\Delta hx,y-\Delta hy,z-\Delta hz}\wedge \quad \textbf{(11)}$$
$$\Delta h_{x}=\Delta h\cos(\Theta)\cos(\sigma),\Delta h_{y}=\Delta h\cos(\Theta)\sin(\sigma),\Delta h_{z}=\Delta h\sin(\Theta)$$

The velocity $\left(\vec{V}_{Tracer}\right)_{x-\Delta hx,y-\Delta hy,z-\Delta hz}$ is not known the velocity known more nearly is: $\left(\vec{V}_{Tracer}\right)_{x-\Delta x,y-\Delta y,z-\Delta z}\wedge\Delta x=\left(u_{Tracer}\right)_{x}\Delta t,\Delta y=\left(v_{Tracer}\right)_{y}\Delta t,\Delta z=\left(w_{Tracer}\right)_{z}\Delta t$. The $\left(\vec{V}_{Tracer}\right)_{x,y,z}$ can be also represented by $\left(\vec{V}_{Tracer}\right)^{t+\Delta t}$. Considering a linear evolution of the tracer velocity along (Δx,Δy,Δz) than equation (11) can be discretized in the follow way:

$$\frac{\Delta h^{t+\Delta t} - \Delta h^{t}}{\Delta t} = \Delta h^{*} \cos(\Theta)\cos(\sigma)\frac{u_{Tracer}{}^{t+\Delta t} - u_{Tracer}{}^{t}}{u_{Tracer}{}^{t+\Delta t}\Delta t} +$$

$$\Delta h^{*} \cos(\Theta)\sin(\sigma)\frac{v_{Tracer}{}^{t+\Delta t} - v_{Tracer}{}^{t}}{v_{Tracer}{}^{t+\Delta t}\Delta t} + \Delta h^{*} \sin(\Theta)\frac{w_{Tracer}{}^{t+\Delta t} - w_{Tracer}{}^{t}}{u_{Tracer}{}^{t+\Delta t}\Delta t} \Leftrightarrow$$

$$\Delta h^{t+\Delta t} = \Delta h^{t} + \Delta h^{*} K \wedge$$

$$K = \cos(\Theta)\cos(\sigma)\frac{u_{Tracer}{}^{t+\Delta t} - u_{Tracer}{}^{t}}{u_{Tracer}{}^{t+\Delta t}} + \cos(\Theta)\sin(\sigma)\frac{v_{Tracer}{}^{t+\Delta t} - v_{Tracer}{}^{t}}{v_{Tracer}{}^{t+\Delta t}} +$$

$$\sin(\Theta)\frac{w_{Tracer}{}^{t+\Delta t} - w_{Tracer}{}^{t}}{u_{Tracer}{}^{t+\Delta t}}$$

To avoid negative lengths an explicit methodology is used when K is positive and a implicit one in the opposite case (12).

$$\Delta h^{t+\Delta t} = \begin{cases} \Delta h^{t}(1+K) \wedge K > 0 \\ \Delta h^{t}/(1-K) \wedge K < 0 \end{cases} \quad (12)$$

Variable Time step

Near the jet input the gradients are very large and a small time steps must be considered but the tendency is the smoothing of these gradients in time due to mixing processes. To increase numerical computation efficiency a variable time step was considered. The criteria use consists in not letting the volume growth due to entrainment ($\Delta V=(E_S+E_D)\Delta t$) in a time step be greater then a percentage (K) of the volume (V). This percentage is defined by the user.

$$\Delta t < \frac{kV^{t}}{E_S + E_D} \bigg/ 100 \quad (13)$$

In the beginning of the simulation is necessary to give a time step to have a volume ($V=Q\Delta t$) because only the flow and the diameter of the port are known.

$$\Delta t = \frac{kV^{t}}{E_S + E_D} \bigg/ 100 = \frac{kQ\Delta t}{\alpha_S \pi D \Delta h \left|\vec{V}_{Tracer} - \vec{V}_{Ambient} \cdot \vec{n}\right| + \alpha_D \pi D \Delta h \left|\vec{V}_{An}\right|} \bigg/ 100 \wedge \Delta h = \frac{Q\Delta t}{\pi \frac{D^2}{4}} \Leftrightarrow$$

$$\Delta t = \frac{kQ\Delta t/100}{\left(\alpha_S \left|\vec{V}_{Tracer} - \vec{V}_{Ambient} \cdot \vec{n}\right| + \alpha_D \left|\vec{V}_{An}\right|\right)\pi D \frac{Q\Delta t}{\pi \frac{D^2}{4}}} \Leftrightarrow$$

The initial time step is computed using equation (14).

$$\Delta t = \frac{k/100}{\left(\alpha_S \left|\vec{V}_{Tracer} - \vec{V}_{Ambient} \cdot \vec{n}\right| + \alpha_D \left|\vec{V}_{An}\right|\right)} \frac{D}{4} \quad (14)$$

Drag and shear Entrainment Parametrizations

The MOHIDJET allows the user to choose between the parametrizations used in the models CORJET and JETLAG to simulate the follow processes: shear entrainment, drag entrainment and drag force. These parametrizations were taken from Lee and Cheung (1990) and Jirka (1999). However some details were inferred and probably this parametrizations are not exactly the ones used by the models. They can be seen was similar. Some work of research must be done to confirm and to explain these differences.

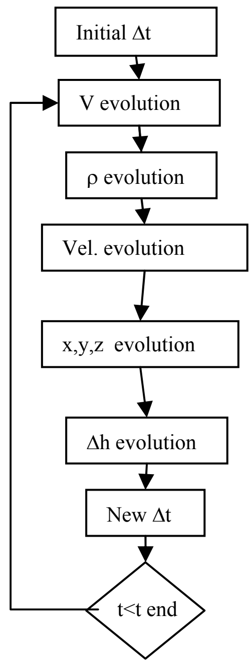
CORJET

Process	Expression
Shear Entrainment	$\alpha_S = 0.055 + \frac{0.6\sin(\theta_{Tracer})}{Fl^2} + 0.055\frac{\lvert\vec{V}_{Ambient}\cdot\vec{n}\rvert}{\lvert\vec{V}_{Tracer}\rvert} \wedge Fl^2 = \max\left(4.66^2, \frac{\lvert\vec{V}_{Tracer}\rvert}{gD\frac{\rho_{Ambient}-\rho_{tracer}}{\rho_{Ambient}}}\right)$ $E_S = \alpha_S \pi \frac{D}{\sqrt{2}} \Delta h \left(\lvert\vec{V}_{Tracer}\rvert - \lvert\vec{V}_{Ambient}\cdot\vec{n}\rvert\right)$
Drag Entrainment	$E_D = \alpha_D \pi \frac{D}{\sqrt{2}} \Delta h \lvert\vec{V}_{An}\rvert \wedge \alpha_D = 0.5$
Drag Force	$\overrightarrow{F_D} = C_D \frac{1}{2} \rho_{Ambient} D \Delta h \lvert\overrightarrow{V_{An}}\rvert \overrightarrow{V_{An}}$

JETLAG

Process	
Shear Entrainment	$\alpha_S = \sqrt{2}\left(0.057 + \frac{0.554\sin(\theta_{Tracer})}{Fl^2}\right)\frac{2\lvert\vec{V}_{Tracer}\rvert}{2\lvert\vec{V}_{Tracer}\rvert - \lvert\vec{V}_{Ambient}\cdot\vec{n}\rvert} \wedge Fl^2 = \max\left(4.66^2, \frac{\lvert\vec{V}_{Tracer}\rvert}{gD\frac{\rho_{Ambient}-\rho_{tracer}}{\rho_{Ambient}}}\right)$ $E_S = \alpha_S \pi D \Delta h \left(\lvert\vec{V}_{Tracer}\rvert - \lvert\vec{V}_{Ambient}\cdot\vec{n}\rvert\right)$
Drag Entrainment	$E_D = \alpha_D D \Delta h \lvert\vec{V}_{An} \wedge \alpha_D = 0.5\rvert$
Drag Force	$\overrightarrow{F_D} = C_D \sqrt{2} \frac{1}{2} \rho_{Ambient} D \Delta h \lvert\overrightarrow{V_{An}}\rvert \overrightarrow{V_{An}}$

MOHIDJET Flowchart



References

Lee, J.H.W. and Cheung, V. (1990) Generalized Lagrangian model for buoyant jets in current. Journal of Environmental Engineering, ASCE, 116(6), pp. 1085-1105.
Jirka, G.H. (1999). "Five Asymptotic Regimes of a Round Buoyant Jet in Stratified Crossflow, 28th IAHR Biennial Congress, Graz (Austria), 23-27 Aug 1999.